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A MODEL OF

MULTINATION MULTIPLANT OPERATION

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RATIONALIZATION IN

MULTINATION MULTIPLANT OPERATION

Abstract

For some industries, the multination multiplant operation means that labor intensive processes are performed in foreign countries where low-cost labor is available. This study develops a linear program model representing such an operation consisting of three production stages and a distribution stage. A solution to the linear program maximizes the sum of net revenues produced by operating units at various stages of operation located in different countries by optimally determining the volumes of goods handled by the units and the sequences of these units. The total cost of operation consists of production and distribution costs, transportation cost, and import duty. The import duty depends on whether the good is a purchased item or a consigned item returning to the owner.

# INTRODUCTION

Labor cost represents a crucial factor of success in market competition for firms producing labor-intensive products. Hence, many of these firms have seriously considered the use of low-cost labor available in other countries. For this reason, some of them have already established or are considering to establish plants in some of the countries in Asia, Latin America, or Southern Europe. In this type of operation, components requiring capital intensive processes are usually produced by plants in more industrialized countries and then shipped to plants in low-cost labor countries for subsequent labor intensive processes such as hand machining, assembly, and testing. Products, thus assembled and tested, are then shipped out of these plants to distribution centers located in more industrialized countries. In this multinational operation, therefore, it is not uncommon to find that all stages of operation are performed in different countries.

This study considers a simple case of the above multination multiplant operation in which only one product line is produced and distributed through a sequence of four stages, including three production stages -- component production, assembly, and test -- and the final stage, or the distribution of completed products. The total operating cost consists of costs for the three production processes, transportation between operating units, transfer of goods or processing of goods under consignment, distribution of finished products, and import duty. Duties charged on imported goods represent a key factor in the analysis of the present operation. To properly evaluate the effects of this factor requires to identify the sequence of all four countries covered by every possible path of goods from the first production stage to the final distribution. The volume of goods passing through each such path is repre-

sented by a variable. The variables thus determined are used to formulate a linear program that represents the present multinational problem with a finite capacity assigned to each operating unit. A solution to this program determines an optimum volume of goods going through each path that maximizes the sum of weighted net revenues produced by the plants and distribution centers. The sum of the volumes going through a particular unit determines the activity level of that unit.

The formulation of the planning model is based on the following considerations that simplify reality without sacrificing the essential feature of the multinational operation:

A. Each country included in the operation is represented by one operating unit which may be a plant, a distribution center, or a combination of these. Thus, the operating unit and the country are synonymous and will be interchangeably used. The capacity of each unit for a specific function, one of production stages or distribution, is given by a finite value.

B. Demand for the finished good in each market is handled by the distribution center in the market and exceeds the capacity of this center. All prices and costs are known, and fixed over the planning period. The price of the finished good depends on the market, whereas the transfer price or the value added under bailment is determined by the stages completed by the good and the two countries involved.

C. The import duty depends on the degree of completion of the good, the value of the good, and the two countries involved. Further, it depends on whether the incoming good is a purchased good or a consigned item returning to its owner. The model is flexible enough to incorporate any complex duty arrangements,

including the regulations of individual countries or the regulations of a common market.

D. A single product is produced and sold in this operation. All units at the same stage of operation make an identical physical contribution toward the finished product. This means, either no local procurement is made, or the same set of components is locally procured and added to the good by all plants at the stage.

It is possible to modify or elaborate the linear program developed in this study in the following aspects:

A. The linear program may be changed from the present maximization model to a model minimizing the total cost required for satisfying given demands in individual markets.

B. A single product line assumed in the study may be changed to a number of product lines. If this change is made, different product lines will compete for services given by the same plant. Further, a single process assumed at each stage may be changed to a set of sub-processes represented by different capacities.

C. With further elaboration, the present single demand period may be changed to a set of sub-periods with separate demands. In this case, the present time-independent program must be changed to a time-dependent program that determines the volume to be handled by each plant or distribution center in each of the sub-periods. Further, the new program may include two additional factors, the time required at each stage of operation and the transfer time between two operating units, but that will greatly increase the complexity of the formulation.

D. Local conditions may be different for different assembly or testing plants and therefore these plants may procure from local sources different sets of components to be added to the goods.

## FORMULATION

The entire cycle of operation is composed of four stages: three production stages -- parts production, assembly, and test -- and distribution, in that sequence. The operating unit located in a country may perform any one or combination of the four stages. In Figure 1, the four operating stages and countries are shown in a matrix form, where a country having a specific function is indicated by a "X". Theoretically, a unit at the preceding stage would be connected to any unit at the following stage, although some of the links would be economically unfeasible.

To represent a particular route of goods from the beginning to the end, we use variable $X_{hijk}$, where subscripts h, i, j, and k represent the numbers of countries associated with the first, second, and third stages of production and the fourth, distribution stage, respectively. For example, $X_{2493}$ means that the three production stages are performed in countries 2, 4, and 9, in that sequence, and the distribution of finished goods in country 3. This particular route is shown by a sequence of arrows in Figure 1. If we let $N_h$, $N_i$, $N_j$, and $N_k$ represent the numbers of countries having operating units in stages 1, 2, 3, and 4, respectively, the total number of possible routes is given by the product $N_hN_iN_jN_k$.

We use H, I, and J, and K to represent sets whose elements are countries that have operating units performing the first, second, third, and fourth stages of operation, respectively. Further, we use h, i, j, or k to represent any country belonging to set H, I, J, or K. Applied to the operating units shown in Figure 1, these symbols give the following relations:

$$
\begin{aligned}
h &\in H = \{1,2,3\} \\
i &\in I = \{1,2,3,4,5,6,7,8\} \\
j &\in J = \{1,2,3,9\} \\
k &\in K = \{1,2,3\}
\end{aligned}
\tag{1}
$$

1. Constraints

We use $e_{an}$ to represent the capacity of a plant that performs work at stage a (a = 1,2,3, or 4) and is located in country n. The volume of goods produced in each of the three stages of production is measured in terms of the number of finished products obtainable from the goods. If wastage is expected at a stage, the volume of goods to be produced at the stage should be increased accordingly so as to produce the number of acceptable finished products at the end.

The total volume of goods going through a unit at each stage should not exceed the capacity of the unit. We write this constraint on the activity level of each unit separately for each stage as follows:

$$\sum_{i\varepsilon I} \sum_{j\varepsilon J} \sum_{k\varepsilon K} X_{hijk} \leq e_{1h} \qquad h \varepsilon H \tag{2}$$

$$\sum_{h\varepsilon H} \sum_{j\varepsilon J} \sum_{k\varepsilon K} X_{hijk} \leq e_{2i} \qquad i \varepsilon I \tag{3}$$

$$\sum_{h\varepsilon H} \sum_{i\varepsilon I} \sum_{k\varepsilon K} X_{hijk} \leq e_{3j} \qquad j \varepsilon J \tag{4}$$

$$\sum_{h\varepsilon H} \sum_{i\varepsilon I} \sum_{j\varepsilon J} X_{hijk} \leq e_{4k} \qquad k \varepsilon K \tag{5}$$

For the program that minimizes the total cost after satisfying demands in individual markets, the sum of goods coming into each distribution center must be at least as big as the demand in this market. Therefore, in this case, we replace constraint (5) by the following (5a):

$$\sum_{h \epsilon H} \sum_{i \epsilon I} \sum_{j \epsilon J} X_{hijk} \geq d_k \qquad k \epsilon K \tag{5a}$$

where $d_k$ is demand for the finished good in country k. Of course, in this case, the demand should not exceed the capacity of the distribution center in the country; that is, $e_{4k} \geq d_k$.

2. Objective Function

The objective of this linear program is to maximize the sum of net revenues after income tax produced by operating plants. The net revenue is the difference between the gross sales and all costs incurred for operation, including the cost of production, transfer price, import duty and transportation cost.

Goods may be transferred permanently or temporarily from a plant in one country to a plant in another country. In the case of permanent transfer, the goods are assumed to be sold at a given transfer price and its entire value is subject to import duty. In the case of temporary transfer, the goods are assumed to be consigned by the primary plant to the secondary plant for additional work under a contract of bailment. In this case, only the value added by the secondary plant is subject to import duty. In order to properly evaluate duties charged on a lot of imported goods, it is essential to keep track of the entire sequence of countries covered by the movement of this lot.

Table 1 lists all possible sequences of countries that goods may go through. In this table, symbols I, II, and III represent no specific

countries but merely to indicate different countries involved. Depending on how different countries appear at different stages, the sequences in Table 1 may be classified into the following four cases:

1. One country appears at all four stages: Sequence 1 belongs to this case where import duties are not assessed.
2. No same country appears at inconsecutive stages: Sequences 2, 4, 5, 10, 11, 14, and 15 belong to this case where goods move from one country to another permanently. Import duties are assessed on the full transfer prices of the goods.
3. The same country appears at inconsecutive stages: Sequences 3, 6, 7, 8, 9, 12, and 13 belong to this case where goods are temporarily transferred from one country to another or others for additional work performed under contract of bailment and then returned to the first country. In this case, import duties are assessed on the values added by consignee plants.
4. Two countries appear alternatingly at consecutive stages: Only sequence [illegible] belongs to this case. Although two different interpretations are [illegible] the consignor-consignee relationship between the [illegible] countries, we assume that country I performing stages 1 and 3 as the consignee, and country II performing stages 2 and 4 as the consignor. Import duties are assessed according to this assumption.

There might be sequences among those listed in Table 1 which would not be practical in reality. Such sequences must be eliminated from consideration when this model is applied to a specific situation. Each sequence in Table 1 will be represented by variable $X_{hijk}$ with subscripts having relationships explained in the right-hand column.

The unit net income function for the plant in a country may be described in two different ways, depending on whether this plant is doing work for itself or for another plant under a contract of bailment. In general, the unit net income for a plant or a distribution center, that acquires goods from another plant or consigns goods to another plant, is given by the following formula:

$$\begin{aligned}\begin{pmatrix}\text{Unit}\\ \text{Net}\\ \text{Income}\end{pmatrix} = \left[1-\begin{pmatrix}\text{Fractional}\\ \text{Rate of}\\ \text{Income Tax}\end{pmatrix}\right]\Bigg[\begin{pmatrix}\text{Unit}\\ \text{Selling}\\ \text{Price}\end{pmatrix} - \left\{1+\begin{pmatrix}\text{Fractional}\\ \text{Rate of}\\ \text{Import Duty}\end{pmatrix}\right\}\begin{pmatrix}\text{Unit Purchase}\\ \text{Price or}\\ \text{Unit Value}\\ \text{Added}\end{pmatrix}\\ - \begin{pmatrix}\text{Unit}\\ \text{Transportation}\\ \text{Cost}\end{pmatrix} - \begin{pmatrix}\text{Unit Cost of}\\ \text{Production and/or}\\ \text{Distribution}\end{pmatrix}\Bigg] \qquad (6)\end{aligned}$$

In (6), if the plant were to process goods at two consecutive stages, the unit purchase price would be zero, and so would be the unit transportation cost. The purchase or selling price here represents the inter-firm transfer price for the buying or selling plant. In general, the purchasing side pays the import duty and shipping cost in addition to the transfer price. In the case of bailment, the owner of the goods pays all the costs incurred that include the values added, transportation costs, and import duties. Thus the net income function for the plant specialized in work at the first stage or the consignee plant doing work at any stage is given by the following, simpler formula:

$$\begin{pmatrix}\text{Unit}\\ \text{Net}\\ \text{Income}\end{pmatrix} = \left[1-\begin{pmatrix}\text{Fractional}\\ \text{Rate of}\\ \text{Income Tax}\end{pmatrix}\right]\left[\begin{pmatrix}\text{Unit Selling}\\ \text{Price or Unit}\\ \text{Value Added}\end{pmatrix} - \begin{pmatrix}\text{Unit Cost}\\ \text{of Produc-}\\ \text{tion}\end{pmatrix}\right] \qquad (7)$$

We assume that the corporate office of world-wide operation assigns different weights to net income values produced in different countries because of long-run considerations such as the transferability of capital,

expected capital facility expansion, and long-run market penetration. In formulating the weighted net income, we use the following terms:

$V_m$ = the weighted total net income, the total net revenue less income taxes, available from the operation using sequence m (m = 1,...,15).

$\alpha_n$ = the relative weight of net income produced in country n, determined on the basis of the [illegible] long-run strategies.

$S_n$ = the fractional rate of income tax in country n.

$U_k$ = the unit selling price of the finished good sold in country k.

$P_{abc}$ = the unit transfer price of the good that has completed stage a and is shipped from country b to country c.

$A_{abc}$ = the value added on the good under bailment at stage a by country b for country c.

$C_{ab}$ = the unit cost of operation charged on the good at stage a in country b.

$E_{abc}$ = the fractional rate of import duty charged on the good that has completed stage a and is imported by country c from country b.

$T_{abc}$ = the unit transportation cost of the good that has completed stage a and is shipped from country b to country c.

The subscripts b and c of $P_{abc}$, $A_{abc}$, $C_{ab}$, $E_{abc}$, or $T_{abc}$ represent the following specific elements for respective stages given by a:

if a = 1, $b = h \in H$ and $c = i \in I$

if a = 2, $b = i \in I$ and $c = j \in J$

if a = 3, $b = j \in J$ and $c = k \in K$

if a = 4, $b = k \in K$

where a = 4 applies only to $C_{ab}$.

The weighted net income function for each sequence in Table 1 is as follows:

<u>Sequence 1</u> $(h=i=j=k)$:

$$V_1 = \sum_{h \in H \cap I \cap J \cap K} \alpha_h (1-S_h)(U_h - C_{1h} - C_{2h} - C_{3h} - C_{4h}) X_{hhhh} \tag{8}$$

<u>Sequence 2</u> $(h=i=j\neq k)$:

$$V_2 = \sum_{h \in H \cap I \cap J} \sum_{\substack{k \in K \\ k \neq h}} [\alpha_h (1-S_h)(P_{3hk} - C_{1h} - C_{2h} - C_{3h})$$

$$+ \alpha_k (1-S_k)\{U_k - (1+E_{3hk})P_{3hk} - T_{3hk} - C_{4k}\}] X_{hhhk} \tag{9}$$

<u>Sequence 3</u> $(h=i=k\neq j)$:

$$V_3 = \sum_{h \in H \cap I \cap K} \sum_{\substack{j \in J \\ j \neq h}} [\alpha_h (1-S_h)\{U_h - (1+E_{3jh})A_{3jh} - T_{2hj} - T_{3jh} - C_{1h} - C_{2h} - C_{4h}\}$$

$$+ \alpha_j (1-S_j)(A_{3jh} - C_{3j})] X_{hhjh} \tag{10}$$

<u>Sequence 4</u> $(h=i\neq j=k)$:

$$V_4 = \sum_{h \in H \cap I} \sum_{\substack{j \in J \cap K \\ j \neq h}} [\alpha_h (1-S_h)(P_{2hj} - C_{1h} - C_{2h})$$

$$+ \alpha_j (1-S_j)\{U_j - (1+E_{2hj})P_{2hj} - T_{2hj} - C_{3j} - C_{4j}\}] X_{hhjj} \tag{11}$$

<u>Sequence 5</u> $(h=i\neq j,\ k\neq j,\ j\neq k)$:

$$V_5 = \sum_{h \in H \cap I} \sum_{\substack{j \in J \\ j \neq h}} \sum_{\substack{k \in K \\ k \neq h, j}} [\alpha_h (1-S_h)(P_{2hj} - C_{1h} - C_{2h}) + \alpha_j (1-S_j)\{P_{3jk}$$

$$- (1+E_{2hj})P_{2hj} - T_{2hj} - C_{3j}\} + \alpha_k (1-S_k)\{U_k - (1+E_{3jk})P_{3jk} - T_{3jk} - C_{4k}\}] X_{hhjk} \tag{12}$$

<u>Sequence 6</u> $(h=j=k\neq i)$:

$$V_6 = \sum_{h \in H \cap J \cap K} \sum_{\substack{i \in I \\ i \neq h}} [\alpha_h (1-S_h)\{U_h - (1+E_{2ih})A_{2ih} - T_{1hi} - T_{2ih} - C_{1h} - C_{3h} - C_{4h}\}$$

$$+ \alpha_i (1-S_i)(A_{2ih} - C_{2i})] X_{hihh} \tag{13}$$

Sequence 7 $(h=j\neq i=k)$:

$$V_7 = \sum_{h\in H\cap J}\ \sum_{\substack{i\in I\cap K\\ i\neq h}} [\alpha_h(1-S_h)(P_{1h}\text{[illegible]}_{3h}-C_{1h}-C_{3h})$$

$$+\ \alpha_i(1-S_i)\{U_i-(1+E_{1h})P_{1hi}-(1+\text{[illegible]})A_{3hi}-T_{1hi}-T_{2ih}-T_{3hi}$$

$$-\ C_{2i}-C_{4i}\}]X_{hihi} \tag{14}$$

Sequence 8 $(h=j\neq i,k;\ i\neq k)$:

$$V_8 = \sum_{h\in H\cap J}\ \sum_{\substack{i\in I\\ i\neq h}}\ \sum_{\substack{k\in K\\ k\neq h,i}} [\alpha_h(1-S_h)\{P_{3hk}-(1+E_{2ih})A_{2ih}-T_{1hi}-T_{2ih}$$

$$-\ C_{1h}-C_{3h}\} + \alpha_i(1-S_i)(A_{2ih}-C_{2i})$$

$$+\ \alpha_k(1-S_k)\{U_k-(1+E_{3hk})P_{3hk}-T_{3hk}-C_{4k}\}]X_{hihk} \tag{15}$$

Sequence 9 $(h=k\neq i=j)$:

$$V_9 = \sum_{h\in H\cap K}\ \sum_{\substack{i\in I\cap J\\ i\neq h}} [\alpha_h(1-S_h)\{U_h-(1+E_{3ih})(A_{2ih}+A_{3ih})$$

$$-\ T_{1hi}-T_{3ih}-C_{1h}-C_{4h}\} + \alpha_i(1-S_i)(A_{2ih}+A_{3ih}$$

$$-\ C_{2i}-C_{3i})]X_{hiih} \tag{16}$$

Sequence 10 $(h\neq i=j=k)$:

$$V_{10} = \sum_{h\in H}\ \sum_{\substack{i\in J\cap J\cap K\\ i\neq h}} [\alpha_h(1-S_h)(P_{1hi}\text{[illegible]})$$

$$+\ \alpha_i(1-S_i)\{U_i-(1+E_{1hi})P_{1hi}-T_{1hi}-C_{2i}-C_{3i}\text{[illegible]}\}]X_{hiii} \tag{17}$$

Sequence 11 $(h\neq i,k;\ i=j\neq k)$:

$$V_{11} = \sum_{h\in H}\ \sum_{\substack{i\in I\cap J\\ i\neq h}}\ \sum_{\substack{k\in K\\ k\neq h,i}} [\alpha_h(1-S_h)(P_{1hi}-C_{1h})$$

$$+\ \alpha_i(1-S_i)\{P_{3ik}-(1+E_{1hi})P_{1hi}-T_{1hi}-C_{2i}-C_{3i}\}$$

$$+\ \alpha_k(1-S_k)\{U_k-(1+E_{3ik})P_{3ik}-T_{3ik}-C_{4k}\}]X_{hiik} \tag{18}$$

Sequence 12 $(h=k\neq i,j;\ i\neq j)$:

$$V_{12} = \sum_{h\in H\cap K} \sum_{\substack{i\in I \\ i\neq h}} \sum_{\substack{j\in J \\ j\neq h,i}} [\alpha_h(1-S_h)\{U_h-(1+E_{3jh})(A_{2ih}+A_{3jh})$$

$$- T_{1hi}-T_{2ij}-T_{3jh}-C_{1h}-C_{4h}\} + \alpha_i(1-S_i)(A_{2ih}-C_{2i})$$

$$+ \alpha_j(1-S_j)(A_{3jh}-C_{3j})]X_{hijh} \tag{19}$$

Sequence 13 $(h\neq i,j;\ i=k\neq j)$:

$$V_{13} = \sum_{h\in H} \sum_{\substack{i\in I\cap K \\ i\neq h}} \sum_{\substack{j\in J \\ j\neq h,i}} [\alpha_h(1-S_h)(P_{1hi}-C_{1h})$$

$$+ \alpha_i(1-S_i)\{U_i-(1+E_{1hi})P_{1hi}-(1+E_{3ji})A_{3ji}$$

$$- T_{1hi}-T_{2ij}-T_{3ji}-C_{2i}-C_{4i}\}$$

$$+ \alpha_j(1-S_j)(A_{3ji}-C_{3j})]X_{hiji} \tag{20}$$

Sequence 14 $(h\neq i,j;\ i\neq j=k)$:

$$V_{14} = \sum_{h\in H} \sum_{\substack{i\in I \\ i\neq h}} \sum_{\substack{j\in J\cap K \\ j\neq h,i}} [\alpha_h(1-S_h)(P_{1hi}-C_{1h})$$

$$+ \alpha_i(1-S_i)\{P_{2ij}-(1+E_{1hi})P_{1hi}-T_{1hi}-C_{2i}\}$$

$$+ \alpha_j(1-S_j)\{U_j-(1+E_{2ij})P_{2ij}-T_{2ij}-C_{3j}-C_{4j}\}]X_{hijj} \tag{21}$$

Sequence 15 $(h\neq i,j,k;\ i\neq j,k;\ j\neq k)$:

$$V_{15} = \sum_{h\in H} \sum_{\substack{i\in I \\ i\neq h}} \sum_{\substack{j\in J \\ j\neq h,i}} \sum_{\substack{k\in K \\ k\neq h,i,j}} [\alpha_h(1-S_h)(P_{1hi}-C_{1h})$$

$$+ \alpha_i(1-S_i)\{P_{2ij}-(1+E_{1hi})P_{1hi}-T_{1hi}-C_{2i}\}$$

$$+ \alpha_j(1-S_j)\{P_{3jk}-(1+E_{2ij})P_{2ij}-T_{2ij}-C_{3j}\}$$

$$+ \alpha_k(1-S_k)\{U_k-(1+E_{3jk})P_{3jk}-T_{3jk}-C_{4k}\}]X_{hijk} \tag{22}$$

The objective function of the present problem is the sum of the weighted net income functions, (8)-(22), defined above for all possible sequences of countries. Thus, it is given by the following W:

$$W = V_1 + V_2 + \ldots + V_{15} \tag{23}$$

With constraints (2)-(5) and the objective function given by (8)-(23), we have completed the linear program maximizing the sum of weighted net income available from all operating units in the multinational operation.

## AN NUMERICAL EXAMPLE

The linear programming model developed in this study is now applied to a numerical example of the case illustrated in Figure 1. Its plants are located in nine countries and their stages of operation and locations were given in (1). Parameters included in the model have the following assumed values:

1. the capacity of each operating unit, $e_{an}$, listed in Table 2,
2. the weight assigned to net income, $\alpha_n$, the tax rate, $S_n$, and the unit price of the finished good, $U_n$, listed in Table 3,
3. the transfer price between two countries, $P_{abc}$, listed in Table 4,
4. the unit value added, $A_{an}$, listed in Table 5,
5. the unit operating cost, $C_{ab}$, listed in Table 6,
6. the rate of import duty, $R_{abc}$, listed in Table 7,
7. the unit transportation cost, $T_{abc}$, listed in Table 8.

In this example, the number of constraint inequalities given in (2)-(5) is 3 + 8 + 4 + 3 or 18, and the number of variables is 3 X 8 X 4 X 3 or 288. This problem was run by an IBM 360/75 using IBM's MPS-X, taking 0.11 minutes and 23 iterations before reaching an optimum solution.

The optimum solution produced the total weighted net income of $16,533 with the following volumes of goods moving through the indicated sequences:

1. 370 units moving through sequence 1-5-9-1.
2. 350 units moving through sequence 1-6-9-1.
3. 70 units moving through sequence 1-7-1-1.
4. 280 units moving through sequence 1-8-1-1.
5. 30 units moving through sequence 1-8-9-1.
6. 150 units moving through sequence 2-4-2-2.
7. 100 units moving through sequence 3-3-9-3.
8. 250 units moving through sequence 3-6-3-3.
9. 100 units moving through sequence 3-7-1-1.

where sequence 1-5-9-1, for example, means that the lot goes through countries 1, 5, 9, and 1 as it progresses from stage 1 through stage 4.

In this solution, the first eight of the nine lots of goods represent cases of bailments; of these, five lots are consigned by the plant in country 1, one lot by the plant in country 2, and three lots by the plant in country 3. The sum of units in the lots terminating in each of the three countries shows the number of finished goods sold in that country. Thus, a total of 1,200 units are sold in country 1, a total of 150 units in country 2, and a total of 350 units in country 3.

FIGURE 3

MATRIX IDENTIFYING STAGES OF OPERATION PERFORMED BY INDIVIDUAL COUNTRIES

| Stage of Operation | Country With Operating Unit | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1. Parts Production | X | X | X | | | | | | |
| 2. Assembly | X | X | X | X | X | X | X | X | |
| 3. Test | X | X | X | | | | | | X |
| 4. Distribution | X | X | X | | | | | | |

TABLE 1

POSSIBLE SEQUENCES OF COUNTRIES INCLUDED IN FOUR-STAGE OPERATION

| Sequence Number | Number of Countries Involved[1] | Existence of Bailment | Countries Involved in Operation[2] Stage 1 | Stage 2 | Stage 3 | Stage 4 | Relationships Between Subscripts of the Variable |
|---|---|---|---|---|---|---|---|
| 1 | One(I) | No | I | I | I | I | h=i=j=k |
| 2 | Two(I,II) | No | I | I | I | II | h=i=j≠k |
| 3 | Two(I,II) | Yes | I | I | II* | I | h=i=k≠j |
| 4 | TWO(I,II) | No | I | I | II | II | h=i≠j=k |
| 5 | Three (I,II,III) | No | I | I | II | III | h=i≠j,k; j≠k |
| 6 | Two(I,II) | Yes | I | II* | I | I | h=j=k≠i |
| 7 | Two(I,II) | Yes | I | II | I* | II | h=j≠i=k |
| 8 | Three (I,II,III) | Yes | I | II* | I | III | h=j≠i,k; i≠k |
| 9 | Two(I,II) | Yes | I | II* | II* | I | h=k≠i=j |
| 10 | Two(I,II) | No | I | II | II | II | h≠i=j=k |
| 11 | Three (I,II,III) | No | I | II | II | III | h≠i,k; i=j≠k |
| 12 | Three (I,II,III) | Yes | I | II* | III* | I | h=k≠i,j; i≠j |
| 13 | Three (I,II,III) | Yes | I | II | III* | II | h≠i,j; i=k≠j |
| 14 | Three (I,II,III) | No | I | II | III | III | h≠i,j; i≠j=k |
| 15 | Four (I,II,III,IV) | No | I | II | III | IV | h≠i,j,k; i≠j,k; j≠k |

Notes: 1. I, II, III, and IV indicate different countries in the sequence of participation in the operation.

2. Countries with * perform work under bailment contract.

Table 2

Capacities of Operating Units, $e_{an}$, in Units of Finished Goods

| | Plant Located in Country n | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Stage of Operation a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1000 | 150 | 3000 | | | | | | |
| 2 | 100 | 70 | 100 | 150 | 370 | 500 | 250 | 310 | |
| 3 | 450 | 150 | 250 | | | | | | 750 |
| 4 | 1100 | 300 | 350 | | | | | | |

Table 3

Weights Assigned to Net Income, $\alpha_n$, Income Tax Rates, $S_n$, and Unit Prices of Finished Goods, $U_n$

| | Plant Located in Country n | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Item | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Weight $\alpha_n$ | 1.0 | .8 | .9 | .4 | 8 | .7 | 1.0 | .7 | .8 |
| Tax Rate $S_n$ | .5 | .4 | .4 | 5 | 3 | .3 | .5 | .6 | .6 |
| Unit Price $U_n$ | 38 | 37 | 36 | | | | | | |

Table 4

Transfer Prices Between Countries, $F_{abc}$, in Dollars

| Stage Completed by Goods a | From Plant in Country b | To Plant in Country c | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | - | 5.7 | 5.6 | 5.6 | 5.6 | 5.4 | 5.4 | 5.5 | |
| | 2 | 5.5 | - | 5.8 | 5.8 | 5.6 | 5.6 | 5.5 | 5.8 | |
| | 3 | 5.4 | 5.3 | - | 5.3 | 5.5 | 5.4 | 5.4 | 5.5 | |
| 2 | 1 | - | 21.0 | 21.4 | | | | | | 21.5 |
| | 2 | 20.2 | - | 20.5 | | | | | | 20.1 |
| | 3 | 20.0 | 19.9 | - | | | | | | 19.8 |
| | 4 | 19.4 | 19.3 | 19.2 | | | | | | 19.3 |
| | 5 | 18.7 | 18.6 | 18.4 | | | | | | 18.0 |
| | 6 | 17.9 | 17.8 | 17.9 | | | | | | 17.8 |
| | 7 | 21.2 | 21.0 | 20.9 | | | | | | 20.8 |
| | 8 | 17.9 | 17.6 | 17.8 | | | | | | 17.8 |
| 3 | 1 | - | 29.0 | 29.2 | | | | | | |
| | 2 | 28.5 | - | 28.4 | | | | | | |
| | 3 | 29.1 | 29.0 | - | | | | | | |
| | 9 | 27.8 | 27.[illegible] | 27.7 | | | | | | |

Table 5

Unit Values Added Under Bailment $A_{abc}$ in Dollars

| Stage Completed by Goods a | Consignee's Country b | Consignor's Country c | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 1 | - | 12.6 | 12.4 | | | | | | 12.7 |
| | 2 | 9.0 | - | 9.5 | | | | | | 9.4 |
| | 3 | 8.6 | 8.5 | - | | | | | | 8.5 |
| | 4 | 8.8 | 8.8 | 8.7 | | | | | | 9.0 |
| | 5 | 7.9 | 8.1 | 8.3 | | | | | | 8.5 |
| | 6 | 5.9 | 5.9 | 5.8 | | | | | | 6.1 |
| | 7 | 9.0 | 8.7 | 8.9 | | | | | | 8.7 |
| | 8 | 7.7 | 7.9 | 7.8 | | | | | | 7.8 |
| 3 | 1 | - | 6.4 | 6.6 | | | | | | |
| | 2 | 5.9 | - | 5.9 | | | | | | |
| | 3 | 5.7 | 5.9 | - | | | | | | |
| | 9 | 4.2 | 4.3 | 4.1 | | | | | | |

Table 6

Unit Operating Costs $C_{an}$ in Dollars

| Stage Completed by Goods a | Plant Located in Country n | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 3 | 3.5 | 3.2 | | | | | | |
| 2 | 10 | 8.0 | 7.0 | 7.4 | 5.0 | 4.0 | 7.0 | 5.0 | |
| 3 | 5 | 4.2 | 4.0 | | | | | | 2.7 |
| 4 | 2 | 2.1 | 2.5 | | | | | | |

Table 7

Rates of Import Duty $E_{abc}$ in Percent

| Stage Completed by Goods a | Exporting Country b | Importing Country c | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | - | 4% | 3% | 4% | 4% | 6% | 4% | 3% | |
| | 2 | 4% | - | 3% | 0% | 4% | 6% | 3% | 3% | |
| | 3 | 3% | 4% | - | 4% | 4% | 4% | 3% | 6% | |
| 2 | 1 | - | 8% | 6% | | | | | | 0% |
| | 2 | 6% | - | 6% | | | | | | 0% |
| | 3 | 6% | 7% | - | | | | | | 0% |
| | 4 | 6% | 0% | 10% | | | | | | 0% |
| | 5 | 6% | 8% | 7% | | | | | | 0% |
| | 6 | 6% | 7% | 7% | | | | | | 0% |
| | 7 | 0% | 8% | 6% | | | | | | 0% |
| | 8 | 4% | 4% | 5% | | | | | | 0% |
| 3 | 1 | - | 16% | 12% | | | | | | |
| | 2 | 12% | - | 12% | | | | | | |
| | 3 | 12% | 14% | - | | | | | | |
| | 9 | 14% | 16% | 12% | | | | | | |

Table 8

Unit Transportation Costs $T_{abc}$ in Dollars

| Stage Completed by Goods a | From Country b | To Country c | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | - | .20 | .33 | .16 | .40 | .36 | .05 | .02 | |
| | 2 | .20 | - | .35 | .05 | .40 | .46 | .17 | .20 | |
| | 3 | .33 | .35 | - | .46 | .13 | .05 | .40 | .33 | |
| 2 | 1 | - | .60 | 1.10 | | | | | | 1.10 |
| | 2 | .60 | - | 1.40 | | | | | | 1.30 |
| | 3 | 1.10 | 1.40 | - | | | | | | .30 |
| | 4 | .50 | .15 | 1.40 | | | | | | 1.40 |
| | 5 | 1.20 | 1.20 | .40 | | | | | | .20 |
| | 6 | 1.10 | 1.40 | .15 | | | | | | .30 |
| | 7 | .15 | .50 | 1.20 | | | | | | 1.20 |
| | 8 | .06 | .60 | 1.00 | | | | | | 1.10 |
| 3 | 1 | - | .75 | 1.25 | | | | | | |
| | 2 | .75 | - | 1.30 | | | | | | |
| | 3 | 1.25 | 1.30 | - | | | | | | |
| | 9 | 1.25 | 1.50 | .40 | | | | | | |